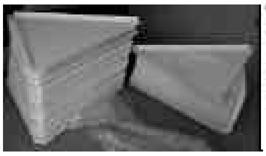
1 Congruent Triangles

Look at these pictures.


Stacks of coins, trays, tiles.

Look at these stacks of books:


Notice the difference?

If you stack up the math textbooks of all kids in your class, will they all fit nicely, one over another?

What about your math notebooks?

The pages of a book all fit exactly over one another.

What about the pages of different books?

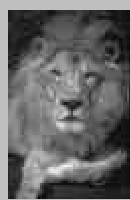
In geometry, figures such as triangles, rectangles, circles and so on, which can be placed one over the other so as to fit exactly are said to be *congruent* figures.

Can you give some examples of congruent and non-congruent figures?

All equal

In geometry, we use the term "congruent" in the sense that "all measures are equal". In the picture on the left, the tiles are of different colors; but all are rectangles and all these rectangles have the same length and breadth. And precisely because of this, they all can be stacked together nicely, each fitting exactly over another. Thus in general, we can say that congruent figures are those with the same shape and size.

For example, two copies of the same photo have the same shape and size; and so they are congruent.

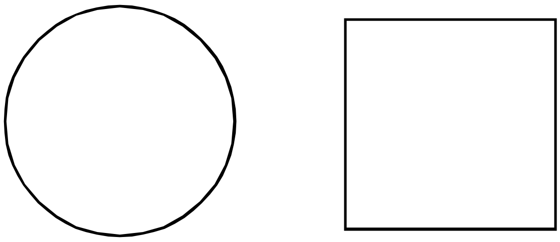



What about a photo and its enlargement?


Even though there is no difference in shape, their sizes are different, aren't they? So, they are not congruent.

Let’s measure

Look at these figures:



A circle and a square. Can you draw figures congruent to these in your notebook?

How do you do it?

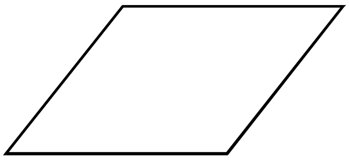
Copy them using tracing paper; or measure the figures and draw with the same measurements.

What are the things to be measured?

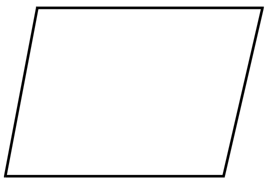
Look at this rectangle:

To draw a rectangle congruent to this, what all things should we measure?

Appu drew a parallelogram:



Ammu measured its sides and drew a parallelogram with the same measurements, like this:



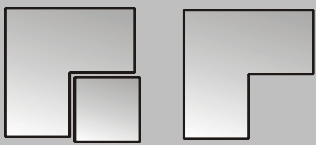
Measure the sides of both parallelograms. The lengths are equal, aren’t they? But we can tell at a glance that these are not congruent.

For parallelograms to be congruent, what more should be equal, apart from the lengths of sides?

Can you draw a parallelogram congruent to the one Appu had drawn?

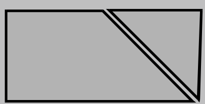
Congruent dissection

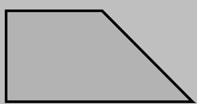
Take a square sheet of paper and cut off a quarter as shown below:



The remaining figure has to be cut into four congruent pieces. Can you do it?

Another puzzle. Cut out a paper rectangle with one side twice as long as the other.

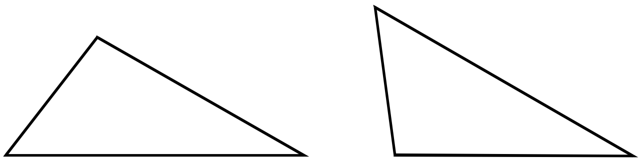




Can you cut this into four congruent figures?

Triangle match

Take a look at these triangles:



Are they congruent?

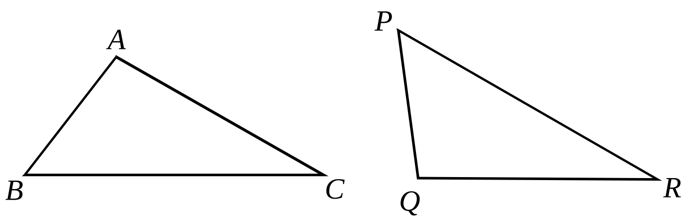
Copy one of these on a piece of tracing paper and place it over the other at different positions.

They are congruent, aren't they?

When the triangles were made to fit exactly over each other, which were the sides that matched?

And which angles?

To specify the equal sides and angles, let's name the triangles.



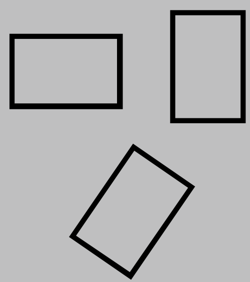
Complete the table, by writing the last pair of equal sides and angles.

Equal sides	Equal angles
$AB = PQ$	$\angle ACB = \angle PRQ$
$BC = PR$	$\angle BAC = \angle PQR$

Do you see any relation between each pair of equal sides and the pair of equal angles given alongside?

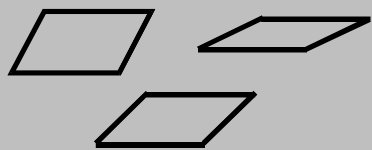
Different angles

Cut out a pair of *eerkkil* bits, each 3 centimeters long and another pair, each 2 centimeters long. How many different kinds of rectangles can you make with these?



We can only place the same rectangle in different positions; we cannot make different types of rectangles.

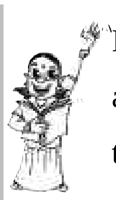
Now with the same *eerkkil* pieces, how many different types of parallelograms can you make?



As many as we want, right?

Now place one of the longer pair of *eerkkil* pieces and one of the shorter pairs, at an angle of 45°, as shown below:

Without shifting these, how many different parallelograms can you make using the other two with these?



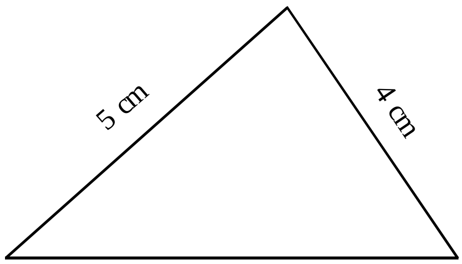
If two triangles are congruent, then the sides and angles of one are equal to the sides and angles of the other; angles opposite to equal sides are equal and sides opposite to equal angles are equal.

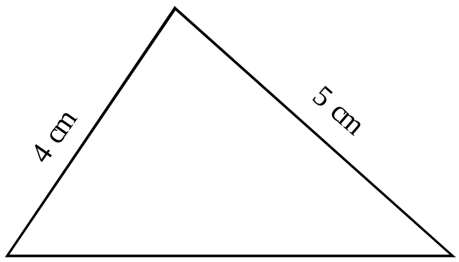
When sides are equal

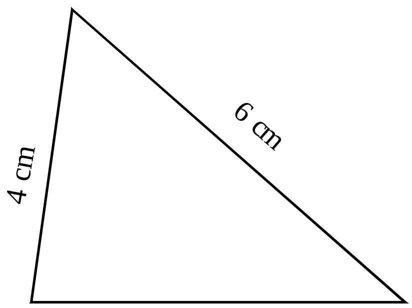
In Class 7, we have seen how we can draw triangles with specified measures (see the lesson, *Math Drawing*).

Can you draw a triangle of sides 4 centimeters, 5 centimeters and 6 centimeters?

Given below are pictures of such triangles which some kids in a class drew.



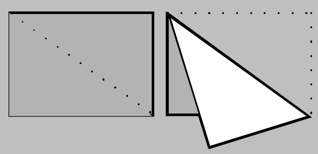




Fold and turn

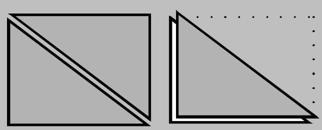
Cut out a paper square. If it is folded along the diagonal, we get two triangles and these triangles fit exactly over each other; that is, these triangles are congruent.

Now cut out a paper rectangle, which is not a square. Again fold through the diagonal.

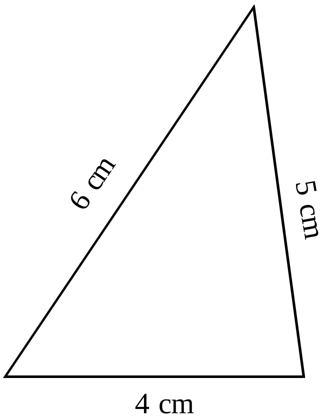


They don't fit over each other. Can we say that these triangles are not congruent?

Cut out these triangles and place one over the other in various positions.



What do you say now?



Make a copy of one of these on a piece of tracing paper and place it over the others. These triangles are all congruent, aren't they?

What do we see here?

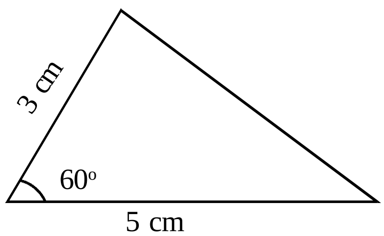
If the three sides of a triangle are equal to the three sides of another triangle, then these triangles are congruent.

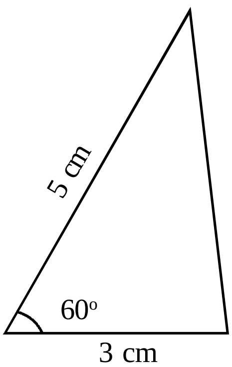
Two sides and an angle

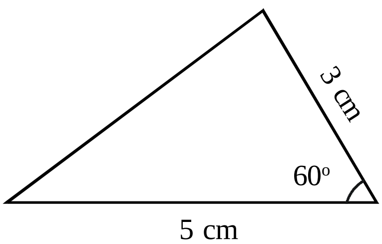
We can draw a triangle, if two sides and the angle included between them are specified, instead of three sides.

For example, can you draw a triangle with two sides 5 centimeters and 3 centimeters long and the angle between them equal to 60°?

We can draw in different ways:



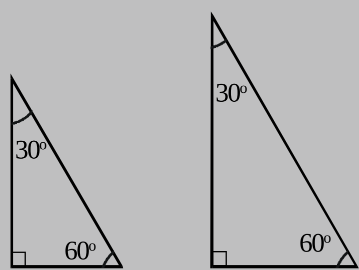




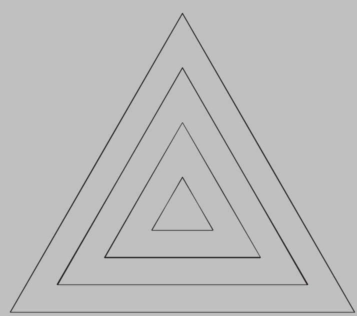
Even if angles are equal

We have noted that if all the sides of a triangle are equal to the sides of another triangle, then these triangles are congruent. What if the angles are equal?

We can draw triangles with the same three angles in different sizes, can't we?



In other words, if the sides of two triangles are equal, their angles are also equal; but simply because angles are equal, the sides need not be equal. Look at this picture:



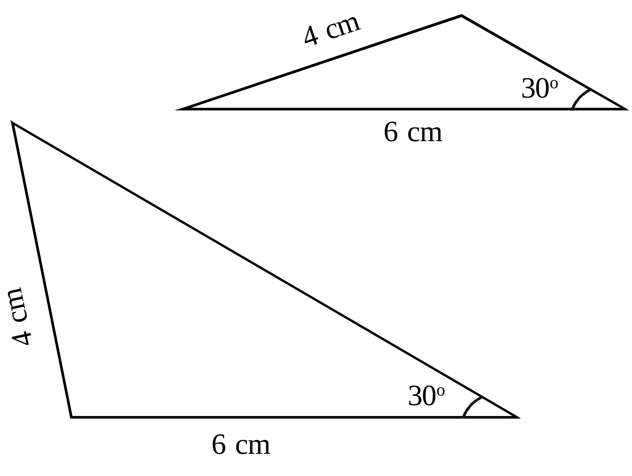
As before, check if these are all congruent to one another. You can also check those drawn by other kids in your class.

What do we see here?

If two sides of a triangle and their included angle are equal to two sides of another triangle and their included angle, then these triangles are congruent.

If some angle other than the included angle is specified, then also we can draw a triangle sometimes.

For example, a triangle with two sides 6 centimeters, 4 centimeters and the angle opposite the shorter side equal to 30^o. Do you remember drawing such triangles in Class 7? (See the section, *Another angle* in the lesson, *Math Drawing*).

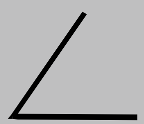


We can tell at a glance that these are not congruent. What do we see here?

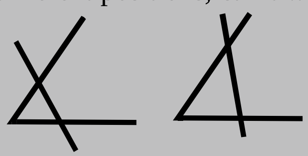
Just because two sides and some angle of a triangle are equal to two sides and some angle of another triangle, the two triangles need not be congruent.

Determining a triangle

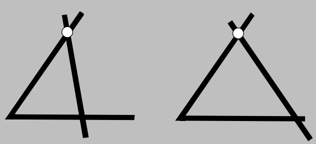
Make an angle by bending a long piece of *eerkkil*.



Now we have to place another piece of *eerkkil* over the two sides of the angle, to make a triangle. We can put it in different positions, can't we?



Let's mark a spot on the top side of the angle. What if we insist that the **second** *eerkkil* should pass through this?



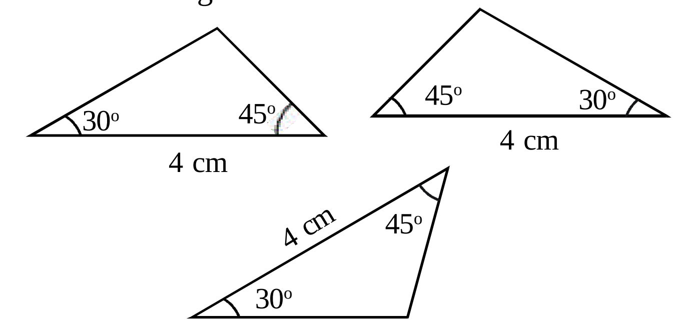
Now let's mark spots on both the top and bottom sides and want the second piece to pass through both these. How many triangles can we make?

Once we fix one angle and the lengths of its two sides, a triangle is determined, isn't it?

One side and two angles

We have seen that we can draw a triangle, if one of the sides and the two angles on it are specified. For example, draw a triangle with one side 4 centimeters long and the two angles on it equal to 30° and 45°.

Some such triangles are shown below:



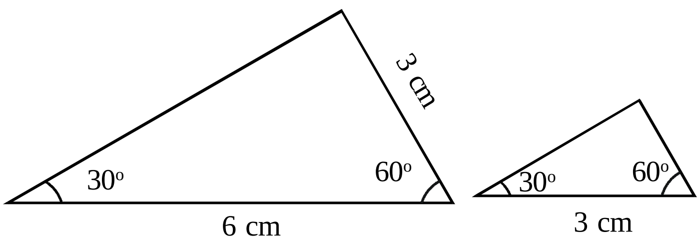
Compare these triangles and also those drawn in your class.

What do you see?

If one side and the two angles on it of a triangle are equal to one side of and the two angles on it of another triangle, then these triangles are congruent.

If two triangles have one side and any two angles equal, is it necessary that they should be congruent?

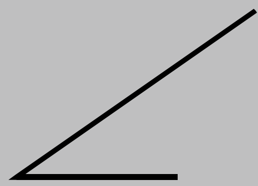
For example, draw a triangle with one side 6 centimeters and the angles on it equal to 30° and 60°. Draw another triangle with one side 3 centimeters and the angles on it again equal to 30° and 60°. Now measure the shortest side of the larger triangle. It is also 3 centimeters, isn't it?



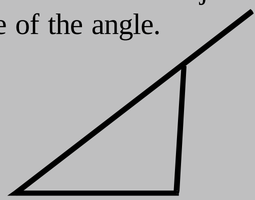
So, in these two triangles, a side (3 centimeters) and two angles (30°, 60°) are equal; but the triangles are not congruent, are they?

How many triangles?

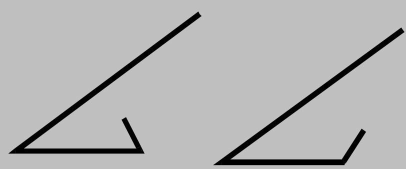
Cut out a 6 centimeter long piece of *eerkkil*. Place a long piece of *eerkkil* at one end of it, making a 30° angle.



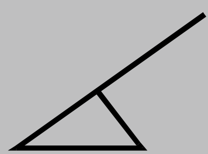
Now we are to make a triangle using another piece of *eerkkil* with this angle. But there are some conditions. One end of this new piece should be at the end of the bottom side of the angle; the other end should just touch the top side of the angle.



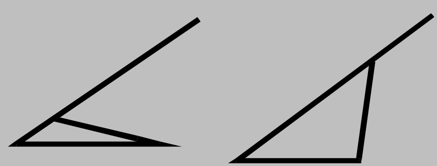
Can we make such a triangle with the third piece 2 centimeters long?



How about a piece 3 centimeters long?



And a 4 centimeter long piece?

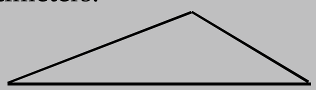


Try with still longer pieces.

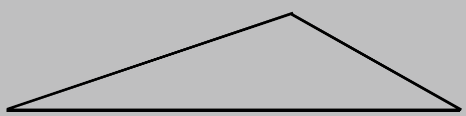
Incorrect match

A triangle has three sides and three angles and thus six measures in all. We have seen that if in two triangles certain triples of these measures (three sides, two sides and the included angle, one side and the angles on them) are equal, then these triangles are congruent; that is, the remaining measures are also equal.

Now take a large sheet of paper and draw a triangle of sides 8, 12 and 18 centimeters.



And then a triangle of sides 12, 18 and 27 centimeters.

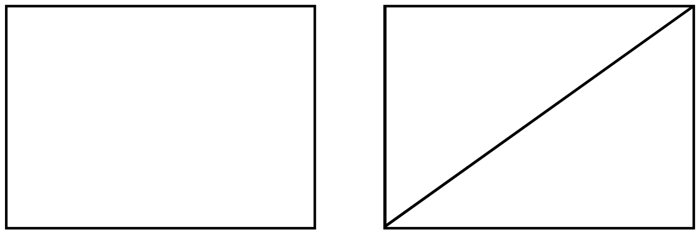


Measure their angles. The angles of the two triangles are equal, aren't they? (You can also check this by cutting out the triangles and placing each angle of one triangle over the angles of the other.)

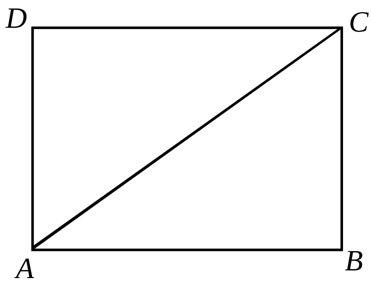
Thus in these two triangles five of the six measures (three angles and two sides) are equal; but they are evidently not congruent.

Applications and examples

When we draw a diagonal of a rectangle, we get two triangles.



And we can see that these two triangles are congruent, by cutting out the triangles and placing one over the other. Now how do we prove that this is so for all rectangles?



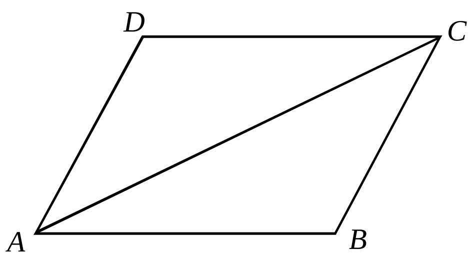
In the rectangle $ABCD$ shown above, the opposite sides AB and CD are equal; and the opposite sides AD and BC are equal.

That is, two sides of ΔACB are equal to two sides of ΔACD. What about the third sides?

In each triangle, the third side is AC. (We can say that AC is a common side of these triangles.)

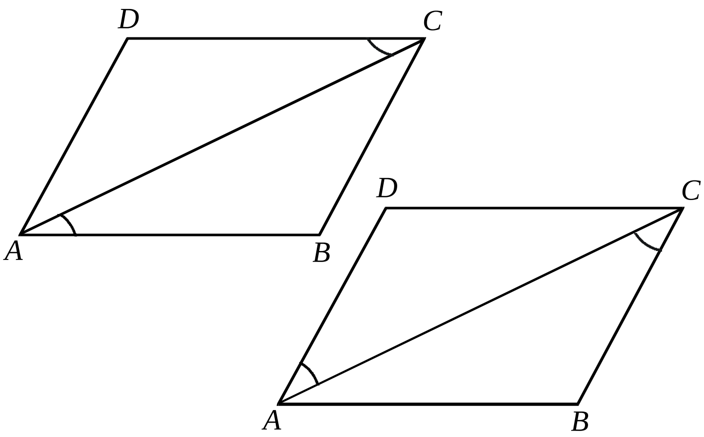
Thus the three sides of ΔACB are equal to the three sides of ΔACD. And so, these triangles are congruent.

What about a parallelogram?



Are the triangles got by drawing a diagonal congruent?

Here also, AC is a common side of ΔACB and ΔACD. But we don't know whether the opposite sides are equal.



Look at the angles on the side AC of each triangle.

$\angle BAC$ and $\angle DCA$ are alternate angles formed by the line AC meeting the pair of parallel lines AB and CD.

So,

$$\angle BAC = \angle DCA$$

Similarly,

$$\angle BCA = \angle DAC$$

being alternate angles formed by the line AC meeting the pair of parallel lines AD and BC. (See the section, *Another kind of pairing*, of the lesson, *Lines in Unison* in the Class 7 textbook.)

Thus, one side of ΔACB and the two angles on it are equal to one side of ΔACD and the two angles on it; and so these triangles are congruent.

We usually use the symbol @ for the phrase "is congruent to". For example, in the figure above,

$$\Delta ACB \cong \Delta ACD$$

The congruency of these two triangles gives another fact:

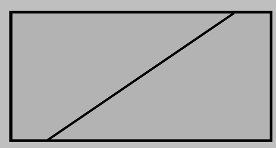
$$AB = CD \text{ and } BC = AD$$

Thus we have the result:

The opposite sides of a parallelogram are equal.

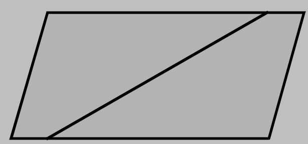
Congruent parts

We saw that by joining a pair of opposite corners of a rectangle, we get two triangles, congruent to each other. Suppose instead of joining opposite corners, we join points equidistant from opposite corners?



Cut these out and check. Draw several such lines. For what position of the line are the pieces also rectangles?

Do parallelograms also have this peculiarity?

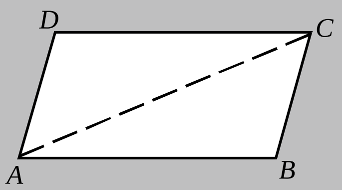


For what position of the dividing line are the pieces also parallelograms?

Parallelogram means...

The general name for a figure of four sides is a *quadrilateral*. A parallelogram is a quadrilateral with both pairs of opposite sides parallel. We proved that in any parallelogram, both pairs of opposite sides are equal. So, there is a reverse question: is every quadrilateral, with both pairs of opposite side equal, a parallelogram?

Suppose in the quadrilateral $ABCD$, we have $AB = CD$ and $AD = BC$. Draw the diagonal AC.



Now we have two triangles ABC and ADC. The sides AB and BC of ΔABC are equal to the sides CD and DA of ΔADC; also the third side of each triangle is AC. Thus the sides of these triangles are equal and so they are congruent.

So, the angles BAC and DCA, which are opposite the equal sides BC and DA, are themselves equal.

These are the alternate angles formed by the line AC with the pair of lines AB and CD. Since these angles are equal, the lines AB and CD are parallel.

Similarly, by drawing the diagonal BD, we can show that $\angle DAC = \angle BCA$ and hence AD and BC are parallel. Thus $ABCD$ is a parallelogram.

What have we proved?

If both pairs of opposite sides of a quadrilateral are equal, then it is a parallelogram.

Again from the congruency of the triangles ACB and ACD in the above discussion, we get

$$\angle ABC = \angle ADC$$

Similarly, by drawing the other diagonal BD, we can prove that ΔBDA and ΔBDC are congruent and hence

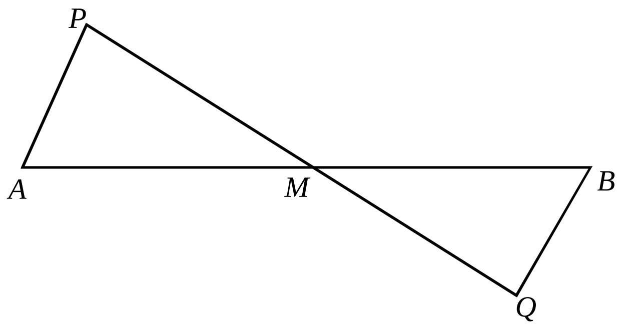
$$\angle BAD = \angle BCD.$$

That is,

The opposite angles of a parallelogram are equal.

Now try these problems:

- In the figure below, AP and BQ are equal and parallel.



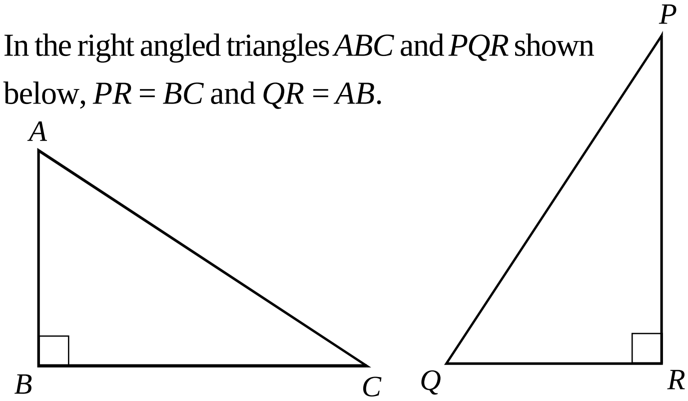
  Prove that $AM = MB$. Can you suggest a construction to locate the midpoint of a line, using this idea?

- Prove that the point of intersection of the two diagonals of a parallelogram is the midpoint of both the diagonals.

Right angled triangles

A right angled triangle is a triangle with one angle right (that is, 90^o), right? The longest side of a right angled triangle is called its hypotenuse. The other two sides may be called the perpendicular sides or the short sides.

In the right angled triangles ABC and PQR shown below, $PR = BC$ and $QR = AB$.



Are they congruent?

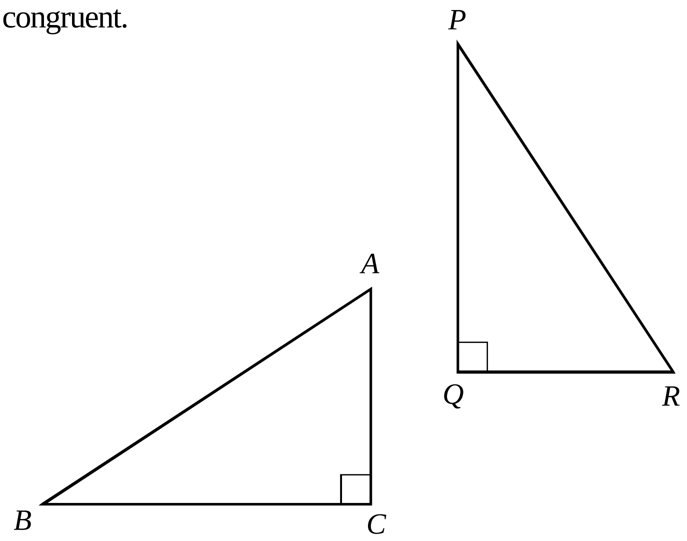
Two sides of ΔPQR are equal to two sides of ΔABC. What more do we need for the triangles to be congruent?

In ΔPQR, the angle PRQ, included between the sides PR and RQ, is a right angle.

In ΔABC, the angle ABC, included between the sides BC and AB is again a right angle. That is,

$$\angle PRQ = 90^{o} = \angle \mathrm{ABC}$$

Thus the sides PR and RQ of ΔPQR and their included angle PRQ are equal to the sides AB and BC of ΔABC and their included angle ABC. So, these triangles are congruent.



If instead of the perpendicular sides, some other pairs of sides are equal, would the right angled triangles be congruent?

In the two right angled triangles shown above, $PR = AB$ and $PQ = BC$.

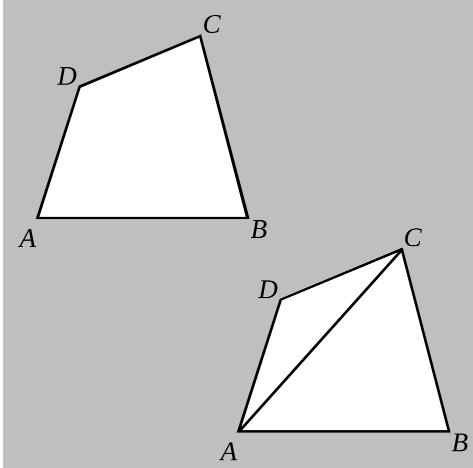
Here we don't know anything about the included angles PRQ and BAC.

Let's take a look at the third sides. Is there any relation between the sides QR and AC?

Angles of a quadrilateral

We saw that in a parallelogram, both pairs of opposite angles are equal. The reverse question is whether a quadrilateral, in which both pairs of opposite angles are equal, is a parallelogram.

To prove that this is true, we must first know the sum of all the four angles of a quadrilateral. Look at these figures:



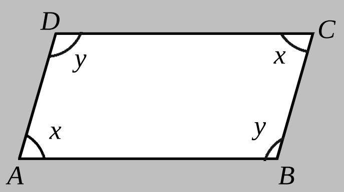
When we draw one diagonal, two angles of the quadrilateral are split into two angles each, giving six angles in all. These six angles are the angles of the ΔABC and ΔACD. So, the sum of these six angles is $180^{o} + 180^{o} = 360^{o}$. What do we see here?

The sum of the angles of a quadrilateral is 360^{o}.

Angles of a parallelogram

Let's see how we can prove that if both pairs of opposite angles are equal in a quadrilateral, then it is a parallelogram.

Consider a quadrilateral with both pairs of opposite angles equal.



In this figure, let's denote the measure of one pair of opposite angles by x^{o} and the measure of the other pair of opposite angles by y^{o}.

Since the sum of the angles of a quadrilateral is 360^{o}, we get

$$x + y + x + y = 360$$

From this we get

$$x + y = 180$$

So, we get

$$\angle A + \angle D = x^{o} + y^{o} = 180^{o}$$

$\angle A$ and $\angle D$ are co-interior angles which the line AD makes with the pair AB, CD of lines. Since their sum is 180^{o}, the lines AB and CD are parallel.

Since we also have $\angle B + \angle D = 180^{o}$, we can prove similarly that AD and BC are parallel.

Thus both pairs of opposite sides are parallel, and so $ABCD$ is a parallelogram.

If both pairs of opposite angles in a parallelogram are equal, then it is a parallelogram.

AB is the hypotenuse of the right angled triangle ABC. So, by Pythagoras Theorem,

$$AC^2 = AB^2 - BC^2$$

Similarly, since PR is the hypotenuse of the right angled triangle PQR, we get

$$QR^2 = PR^2 - PQ^2$$

Now by what we have said at the beginning,

$$PR = AB, \text{ and } PQ = BC$$

Putting all these together, we get

$$QR^2 = PR^2 - PQ^2 = AB^2 - BC^2 = AC^2$$

From this, we find

$$QR = AC$$

Thus the three sides of ΔPQR are equal to the sides of ΔABC. So,

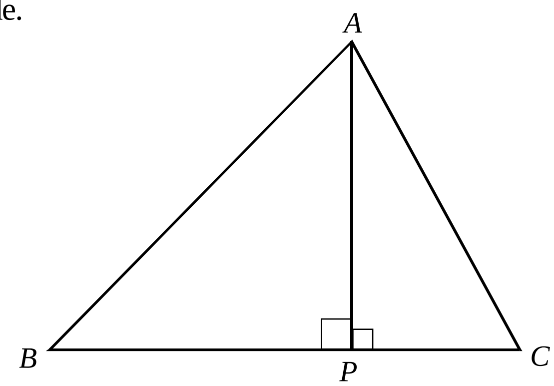
$$\Delta ABC \cong \Delta PQR$$

What general result do we have here?

If the hypotenuse and one other side of a right angled triangle are equal to the hypotenuse and one other side of another right angled triangle, then these two triangles are congruent.

Isosceles triangles

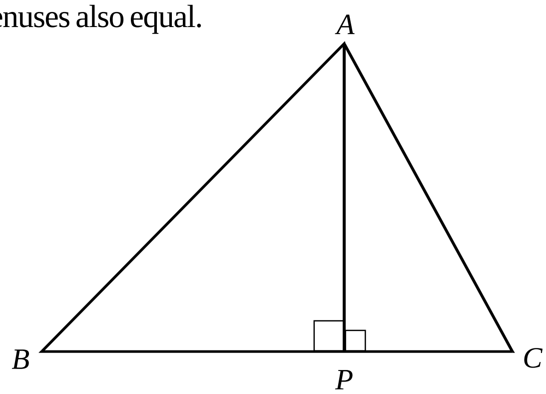
Any triangle can be split into two right angled triangles, by drawing a perpendicular from one vertex to the opposite side.



In the picture above, ΔABC is split into two right angled triangles *ABP* and *ACP*, by drawing the perpendicular from *A* to *BC*.

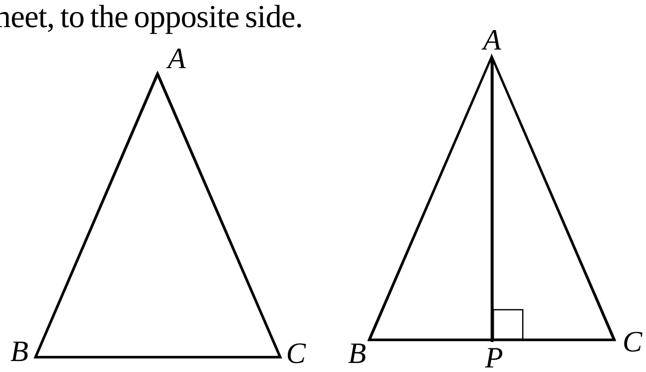
What type of triangle should *ABC* be, so that these two right angled triangles are congruent?

In ΔABP and ΔACP, the side *AP* is common. For these right angled triangles to be congruent, we must have the hypotenuses also equal.



That is, we must have $AB = AC$.

So, a triangle with two of its sides equal can be split into two congruent right angled triangles; we need only draw the perpendicular from the vertex where the equal sides meet, to the opposite side.



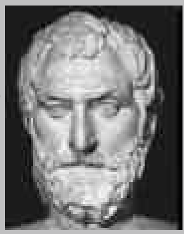
We get another thing from this. In the figure, $AB = AC$ in ΔABC. The line *AP* is drawn perpendicular to the side *BC*. As seen above, the right angled triangles *ABP* and *ACP* are congruent. So, their sides and angles must be equal. For example, since *AP* is a side of both the triangles, the angles opposite this side in the two triangles must be equal. That is,

$$\angle ABC = \angle ACB$$

What do we see here?

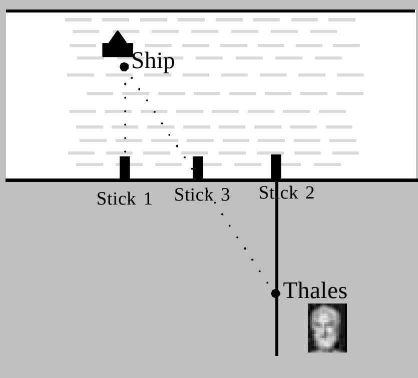
Congruency in action

Thales was a mathematician and philosopher who lived in Greece in the sixth century BC. Here's is a trick he is supposed to have used to calculate the distance to a ship anchored at sea from the shore.



First he stuck a long pole on the shore, directly in front of the ship. Then he stuck another pole on the shore, some distance away from the first one. A third pole he stuck exactly at the middle of the first two poles.

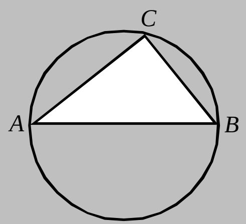
He then drew a line from the second stick, perpendicular to the shore. He walked backwards along this line, keeping the ship in sight. Just when the middle stick came between the ship and himself in the line of sight, he stopped and marked his position.



Now the triangle on sea and the triangle on shore in the picture are congruent. (Why?) So, the distance from the shore to the ship is equal to the distance between the spot where he stopped and the second stick.

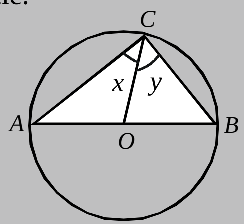
Angle in a semicircle

In the figure below, AB is a diameter of the circle and C is a point on the circle.

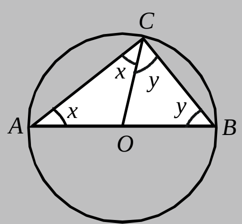


Can we say anything about $\angle ACB$?

For this, join C with the center O of the circle.



Now $\angle ACB$ is split into two. Let the measures of these angles be x^o and y^o. In ΔOAC, we have $OA = OC$. (Why?) So, $\angle OAC = x^o$. Similarly, in ΔOBC, we have $OB = OC$ and so $\angle OBC = y^o$.



The angles of ΔABC are x^o, y^o, $(x + y)^o$. So, $x + y + (x + y) = 180$ and hence $x + y = 90$. Thus, $\angle ABC = 90^o$

In this argument, the point C can be anywhere on the circle. So, what do we see?

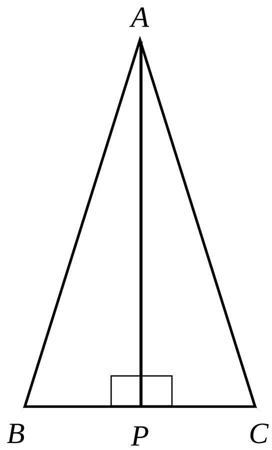
In a circle, the angle formed by joining the endpoints of a diameter with another point on the circle, is a right angle.

If two sides of a triangle are equal, then the angles opposite to these sides are also equal.

Is the reverse true?

That is, if two angles of a triangle are equal, are the sides opposite them also equal?

In ΔABC shown below, $\angle ABC = \angle ACB$. Let's see whether $AB = AC$. As before, we draw the perpendicular from A to BC.



Are the triangles ABP and ACP congruent?

For these triangles, AP is a common side. And one angle on this side is 90^o in both triangles. What more do we need to claim that the triangles are congruent?

Are $\angle BAP$ and $\angle CAP$ equal?

Since the sum of the angles of any triangle is 180^o, we have in ΔABP

$$\angle ABP + \angle BAP + 90^o = 180^o$$

from which we get $\angle ABP + \angle BAP = 90^o$ and hence

$$\angle BAP = 90^o - \angle ABP$$

Similarly, from ΔACP, we get

$$\angle CAP = 90^o - \angle ACP$$

If we use the fact that $\angle ABP = \angle ACP$ also, we get

$$\angle BAP = \angle CAP$$

Thus the side AP of ΔABP and the angles APB and BAP on it are equal to the side AP of ΔACP and the angles APC and CAP on it. So, the triangles are congruent.

And because of this, the sides AB and AC, opposite the equal angles APB and APC are equal.

What do we find?

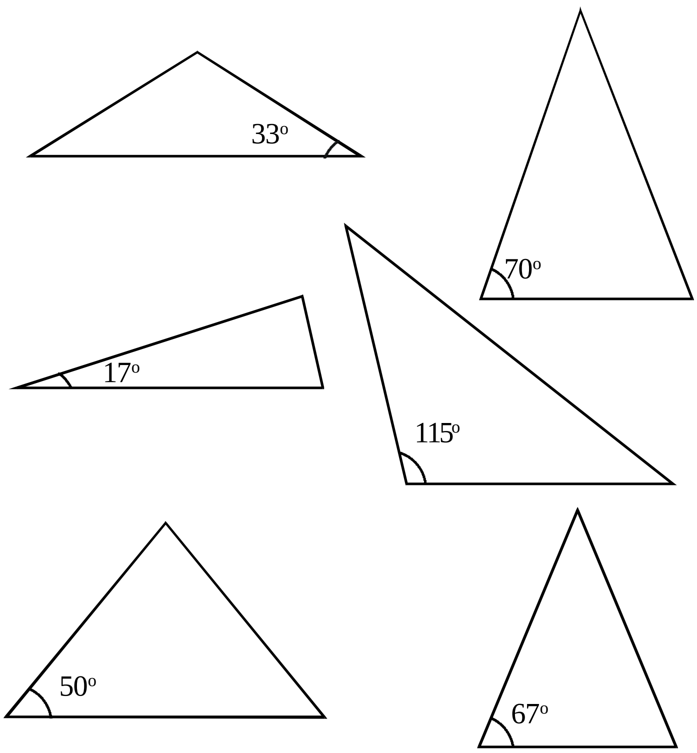
If two angles of a triangle are equal, then the sides opposite these angles are also equal.

A triangle in which two sides are equal is called an *isosceles triangle*. From what we have seen just now, we can also say that an isosceles triangle is a triangle with two angles equal.

Recall that a triangle with all the three sides equal is called an equilateral triangle. Such triangles are a special class of isosceles triangles.

Now you can try some problems:

- Some isosceles triangles are shown below. One angle of each is given. Find the other two angles of each.



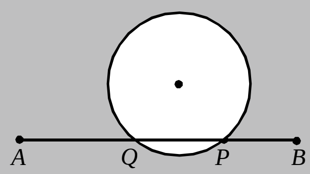
Circle and perpendicular

We are given a line AB and a point P on it.

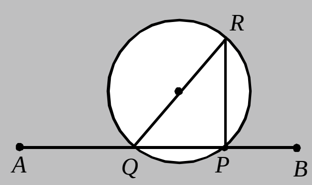
The problem is to draw a perpendicular to AB, passing through P.

We can draw it using a set square in the geometry box. We can also draw it using compass.

For this we draw a circle, which passes through P and cuts AB at another point. We name this point Q.



Now draw the diameter through Q and join its other end with P.



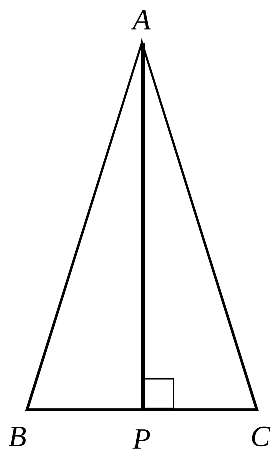
Since QR is a diameter and P is a point on the circle, QPR is a right angle. In other words, PR is perpendicular to AB.

- One angle of an isosceles triangle is 120^o. What are the other two angles?
- What are the angles of an isosceles right angled triangle?
- What are the angles of an equilateral triangle?

Bisectors

We have seen how an isosceles triangle can be divided into two congruent right angled triangles. From this, we can learn a bit more.

In ΔABC shown below, we have $AB = AC$; and AP is the perpendicular from A to BC.



Since ΔABP and ΔACP are congruent, their sides and angles are equal. We get $BP = CP$; also, we get $\angle BAP = \angle CAP$, since these are angles opposite to equal sides.

Thus, the line AP divides the side BC into two equal parts; it divides $\angle BAC$ also into two equal parts.

In an isosceles triangle, the perpendicular from the vertex joining the equal sides to the opposite side, bisects this side and the angle at this vertex.

A line which divides a line or an angle into two equal parts is called a bisector. Thus in the figure above, the line AP is a bisector of the line BC; since it is perpendicular to BC, it is called the perpendicular bisector of BC. It is also the bisector of $\angle BAC$.

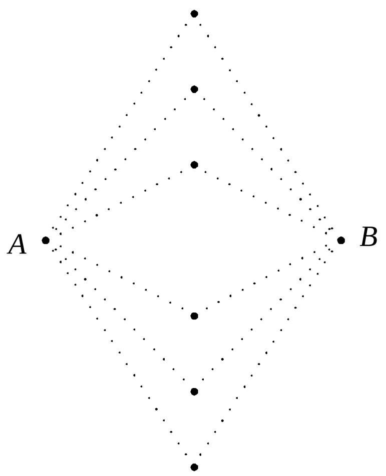
Rope math

We have mentioned the *Elements*, the authorative text on ancient geometry. In this, Euclid considers only figures that can be drawn with straight lines and circles; in other words, only figures that can be drawn with a straight rod without markings and a compass. Why is this so?

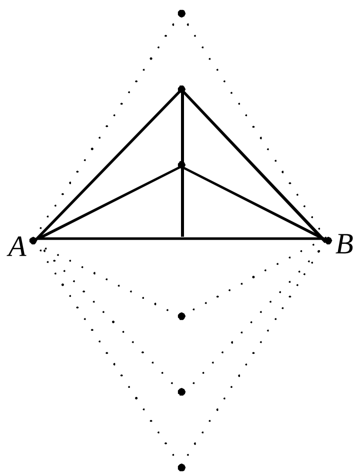
In ancient times, ropes or strings were used for measuring as well as drawing. The two figures that can be easily drawn with rope are straight lines and circles. Lines could be drawn by stretching a rope between two fixed pegs; circles could be drawn by taking out one peg and rotating about the other.

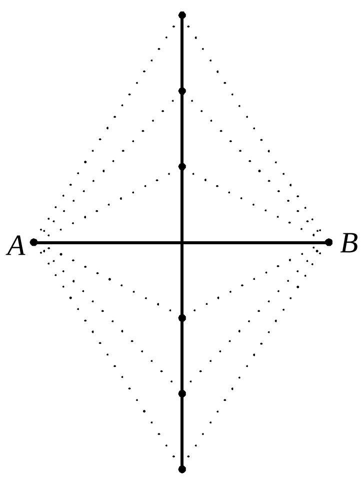
Nowadays, all sorts of tools can be made to draw different kinds of figures and so, such restriction on tools has only historic and theoretical significance.

Now look at this picture:



Several points equidistant from two points *A* and *B* are marked. Each of these points joined to *A* and *B* gives an isosceles triangle. So all these points lie on the perpendicular bisector of the line *AB*. In other words, if we join all these, we get the perpendicular bisector of *AB*.



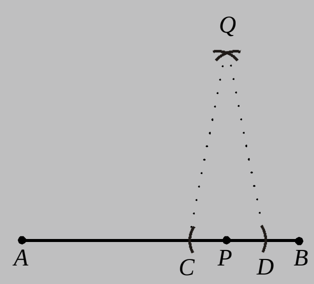


Another way

There is another method to draw the perpendicular to a line from a specified point on it.

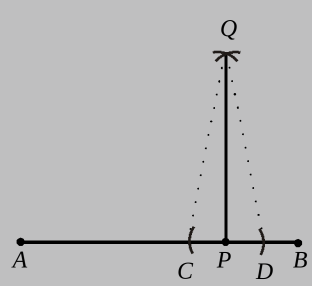
First mark two points *C* and *D* on *AB* itself, at equal distances from *P*.

Next mark a point *Q* equidistant from *C* and *D*.



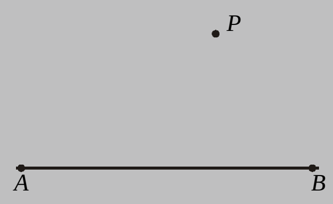
Since *CQD* is an isosceles triangle, the perpendicular from *Q* to *CD* bisects *CD*. This means, this perpendicular passes through the midpoint *P* of *CD*.

In other words, the line *QP* is perpendicular to *CD*. Since the line *CD* is a part of the line *AB*, the line *QP* is perpendicular to *AB* also.

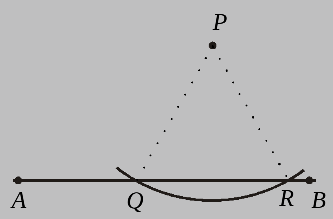


A perpendicular from outside

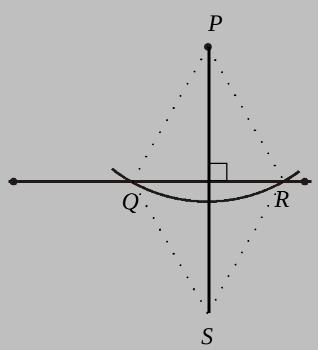
Through a point on a line, we can draw the perpedicular, using a compass. How do we draw the perpendicular from a point not on the line?



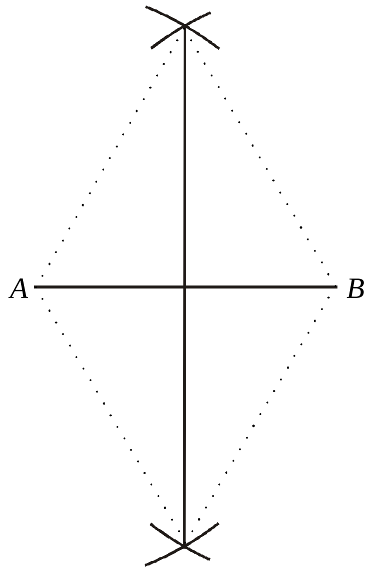
We have to draw an isosceles triangle with *P* as the top vertex and *AB* as the bottom side. For this, we need only mark two points on *AB*, equidistant from *P*. For this draw a circle with *P* as center to cut *AB* at *Q* and *R*.



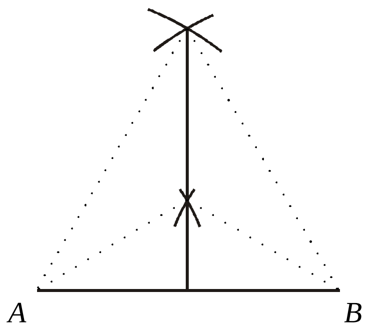
Now we need only draw the perpen-dicular bisector of *QR*. And for that, we need only draw circles of the same radius, with *Q* and *R* as centre.



We need only two points to draw a line. So, we can draw the perpendicular bisector of a line *AB* as shown below:

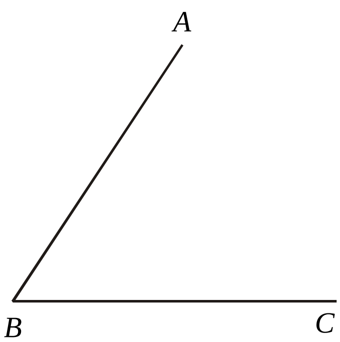


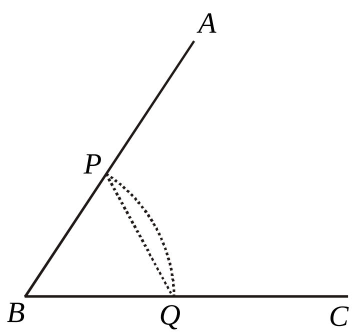
If there is not enough space below *AB*, we can also draw like this:



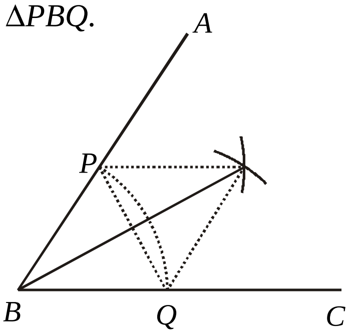
We can use the general result we saw above to draw the bisector of an angle also.

First, we draw an isosceles triangle which contains this angle:



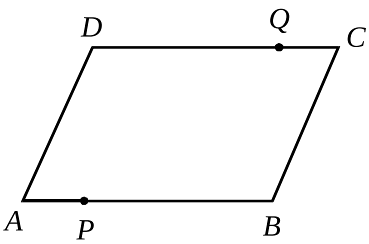


Now we need only draw the perpendicular bisector of the side PQ of ΔPBQ.



Here there's a convenience. The perpendicular bisector we want to draw passes through B (why?) So, we need only mark one more point on this bisector.

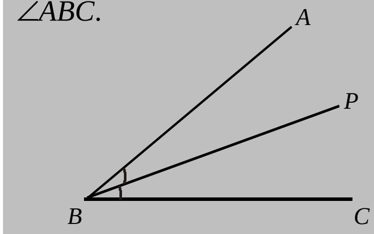
Now try these problems:

- Prove that in a parallelogram with all four sides equal, the diagonals are perpendicular bisectors of each other.
- Can you draw a line of length 2.25 centimeters using a ruler? How about using ruler and compass?
- How can we draw a circle with a given line as diameter, without actually measuring the line?
- How do we draw an angle of $22\frac{1}{2}^{o}$?
- 

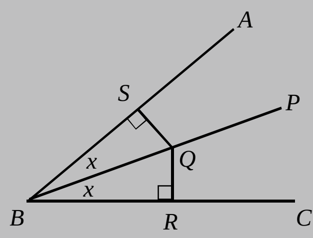
  In the figure, $ABCD$ is a parallelogram and $AP = CQ$. Prove that $PD = BQ$. Prove also that the quadrilateral $PBQD$ is a parallelogram.
- Prove that if one pair of opposite sides of a quadrilateral are equal and parallel, then it is a parallelogram.

Equidistant bisector

In the figure, PQ is the bisector of $\angle ABC$.



Mark a point Q on BP and draw perpendiculars from Q to AB and BC.



Since BP is the bisector of $\angle ABC$, we have $\angle ABP = \angle CBP$.

If we take this as x^o, then

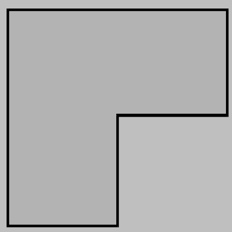
$\angle BQS = \angle BQR = 90^o - x^o$ (Why?)

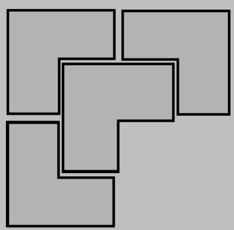
Now we can prove that ΔBQS and ΔBQR are congruent. (How?)

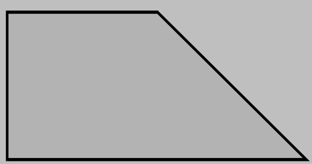
So $QS = QR$.

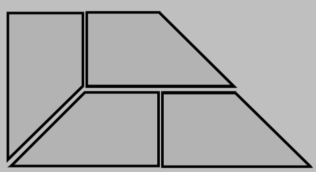
The perpendiculars from a point on the bisectors of an angle, to the sides of the angle, are equal.

Congruent dissections

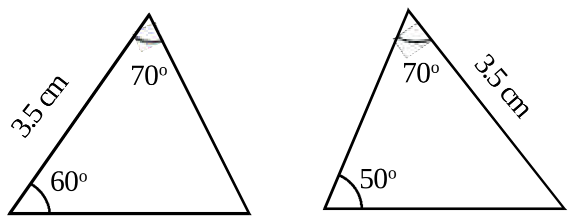








- Are the triangles below congruent? Give the reason.



- How many different (non-congruent) isosceles triangles can be drawn with one angle 80° and one side 8 centimeters?
- In the figure below, $PQ = PR$. Prove that the point P is on the bisector of $\angle ABC$.