



# Negative Numbers

## A card game

A game is played with black and white cards. Each player takes any ten cards and then subtracts the number of black cards from the number of white cards. (Remember playing this game in the lesson *Negative Numbers* in class 7)

For example, suppose one gets 6 white cards and 4 black cards. Then the number got is

$$6 - 4 = 2$$

What if the situation is reversed and one gets 4 white cards and 6 black cards? The number is

$$4 - 6 = -2$$

Now can you complete this table?

White cards	Black cards	Number
7	3	$7 - 3 = 4$
5	5	
3	7	
2	8	
1	9	
0	10	

What is the largest number you got?

And the smallest?

What is the largest number possible in this game?

And the smallest?

## Negative temperature

We know that the coldest place on earth is Antarctica and that temperature there is sometimes as low as  $-89^{\circ}\text{C}$

The coldest place in India is the small town of Dras, in the Kargil region of Kashmir.



Here the temperature sometimes is  $-60^{\circ}\text{C}$ .

In our own Keralam also, high places such as Munnar sometimes record negative temperatures.

### Absolute zero

We have noted that the coldest temperature felt on the earth is  $-89^{\circ}\text{C}$ . The coldest temperature in the whole known universe is found at the “Boomerang nebula”, which is about 500 crore crore ( $5 \times 10^{16}$ ) kilometers from the earth. It is  $-272.15^{\circ}\text{C}$ .



This is the lowest temperature occurring in nature; but lower temperatures are produced artificially in laboratories.

But according to the laws of Physics, temperature below  $-273.15^{\circ}$  celsius just cannot occur. It is called the absolute zero temperature.

### Negative multiplication

Another game with black and white cards. The cards are all shuffled and kept face down. Players draw one card in turn. A white card adds 10 points to the current score, while a black card subtracts 5 points from the score. The first to get 25 points (should be exactly 25) wins.

After five rounds, some hold cards like this:

Name	Black	White
Ammu	3	2
Appu	2	3
Ramu	1	4
Jamila	4	1

Can you find their scores?

Also find how many points each needs to win.

Ammu first got a white and then a black. What was her score then?

$$10 - 5 = 5$$

This can be put another way: a black card gives  $-5$  points and so Ammu's score is

$$10 + (-5) = 10 - 5 = 5$$

Appu got both black in the first two draws. The first draw gave him  $-5$  points. And then what happened to his score when he drew the next black?

$$-5 - 5 = -(5 + 5) = -10$$

Appu's score can be calculated like this also:

$$(-5) + (-5) = -(5 + 5) = -10$$

Now we write  $5 + 5$  as  $2 \times 5$ . Like this, we can also write

$$2 \times (-5) = (-5) + (-5) = -10$$

We have seen that

$$(-5) + (-5) = -(5 + 5)$$

Now, we can write this as

$$2 \times (-5) = -(5 + 5) = -(2 \times 5) = -10$$

So, what is the meaning of  $3 \times (-5)$ ?

$$3 \times (-5) = -(3 \times 5) = -15$$

Some such multiplications are given below. Can you work them out in your head?

- $3 \times (-4)$       •  $4 \times (-3)$       •  $2 \times \left(-\frac{1}{2}\right)$
- $6 \times \left(-\frac{1}{3}\right)$     •  $20 \times \left(-\frac{1}{5}\right)$     •  $5 \times (-0.3)$
- $20 \times (-0.4)$     •  $9 \times (-0.1)$

Two  $-5$ 's make  $-10$ , right? So, we can say that  $-5$  is half of  $-10$ . In mathematical language,

$$\frac{-10}{2} = -5$$

Halving is the same as multiplication by  $\frac{1}{2}$ , isn't it? So,

$$\frac{1}{2} \times (-10) = -5$$

Also,  $5 = \frac{1}{2} \times 10$ . Thus we can write

$$\frac{1}{2} \times (-10) = -\left(\frac{1}{2} \times 10\right) = -5$$

In the same fashion,

$$\frac{1}{3} \times (-12) = -\left(\frac{1}{3} \times 12\right) = -4$$

$$\frac{1}{5} \times (-2) = -\left(\frac{1}{5} \times 2\right) = -\frac{2}{5}$$

Now some more problems to do in your head.

- $\frac{1}{2} \times (-8)$                       •  $\frac{1}{3} \times (-3)$
- $\frac{1}{4} \times \left(-\frac{1}{2}\right)$                       •  $0.5 \times (-0.3)$

All these we can condense into a single algebraic statement:

$$x \times (-y) = -(xy), \text{ for all positive numbers } x \text{ and } y.$$

### Creation of numbers

Different types of numbers are created in mathematics, according to the needs of different ages. During the time when men needed numbers only for counting—the number of people in a group, the number of cattle in a herd—natural numbers were enough. But at a later time when men needed to measure lengths, areas and volumes, fractions were created.

Negative numbers were first used as a convenient tool in some computations. In class 7, we have seen that in ancient China black rods were used for addition and red rods for subtraction; also in ancient India, negative numbers were used to denote debts in problems related to monetary transactions.

Brahmagupta used negative numbers to give general algebraic methods of solving certain problems. This became the major use of negative numbers in later years.

Thus unlike natural numbers and fractions, negative numbers and operations with them evolved not through real life problems, but as convenient algebraic techniques.

### Meaning of operations

For natural numbers, multiplication means repeated addition. For example,

$$2 \times 3 = 3 + 3$$

We can give the same meaning to the product of a fraction by a natural number also. For example,

$$2 \times \frac{1}{3} = \frac{1}{3} + \frac{1}{3}$$

But we cannot define  $\frac{1}{3} \times 2$  as repeated addition. Here we define

$$\frac{1}{3} \times 2 = 2 \times \frac{1}{3}$$

(The product of two natural numbers is the same, whatever be the order of multiplication, isn't it?)

But a product of two fractions, such as  $\frac{1}{2} \times \frac{1}{3}$  cannot be defined like this. Here, we define  $\frac{1}{2} \times \frac{1}{3}$  as  $\frac{1}{2}$  of  $\frac{1}{3}$ . ( $\frac{1}{3} \times 2$  also gives  $\frac{1}{3}$  of 2, doesn't it?).

Similarly, though we can define  $2 \times (-3)$  as repeated addition, this does not have a meaning for  $(-2) \times 3$ . We make a new definition

$$(-2) \times 3 = 3 \times (-2)$$

We have to give yet another definition for  $(-2) \times (-3)$ .

### Order in multiplication

We know that  $2 \times 3$  and  $3 \times 2$  both give 6; that is, adding two 3's or three 2's give the same result, 6. In mathematical symbols,

$$2 \times 3 = 3 \times 2$$

Like this, are  $2 \times (-3)$  and  $(-3) \times 2$  equal?

Let's first see what each of these means.  $2 \times (-3)$  means adding two  $-3$ 's; that is  $(-3) + (-3)$ . But what is  $(-3) \times 2$ ? There is no meaning in saying "minus three 2's added together".

We have to give a new meaning to this operation. In the multiplication of natural numbers, we can take numbers in any order (for example,  $2 \times 3 = 3 \times 2$ ); so in this type of products also, it is convenient to have things that way.

So, we *define*  $(-3) \times 2$  as  $-6$ ; that is

$$(-3) \times 2 = 2 \times (-3) = -6$$

In general, we define

$$(-x) \times y = -(xy), \text{ for all positive numbers } x, y.$$

With this rule, can't you do these in your head?

- $(-3) \times 4$       •  $(-4) \times 7$
- $(-1) \times 9$       •  $(-5) \times 4$
- $(-2) \times 5$       •  $\left(-\frac{1}{3}\right) \times 3$
- $\left(-\frac{1}{4}\right) \times \frac{1}{3}$       •  $\frac{1}{5} \times \left(-\frac{1}{3}\right)$
- $\left(-\frac{1}{4}\right) \times \frac{1}{7}$       •  $(-0.5) \times 0.2$
- $(-0.4) \times 0.3$

## Sums and products

How much is  $(3 \times 12) + (7 \times 12)$ ?

There is an easier way than finding the two products and then adding, remember? (See the mental arithmetic problems at the end of the lesson, *Shorthand Math* in class 7.)

$$\begin{aligned}(3 \times 12) + (7 \times 12) &= (3 + 7) \times 12 \\ &= 10 \times 12 \\ &= 120\end{aligned}$$

Can you do these in your head like this?

$$\begin{aligned}&\bullet (13 \times 8) + (7 \times 8) && \bullet (15 \times 9) + (5 \times 9) \\ &\bullet \left(2 \times \frac{1}{7}\right) + \left(3 \times \frac{1}{7}\right) && \bullet \left(\frac{1}{7} \times 2\right) + \left(\frac{1}{7} \times 5\right)\end{aligned}$$

What is the general principle that we used in all these?

$$xz + yz = (x + y)z \text{ for all positive numbers } x, y, z$$

Can we do  $(3 \times (-12)) + (7 \times (-12))$  like this?

This means, first add  $-12$  three times and then add  $-12$  seven times and finally add these. Instead we can add  $-12$  ten times at one stroke, right?

That is,

$$\begin{aligned}(3 \times (-12)) + (7 \times (-12)) &= (3 + 7) \times (-12) \\ &= 10 \times (-12) \\ &= -120\end{aligned}$$

What about  $\left(\frac{1}{4} \times (-13)\right) + \left(\frac{3}{4} \times (-13)\right)$ ?

What does this mean? First find  $\frac{1}{4}$  of  $-13$ , then  $\frac{3}{4}$  of  $-13$  and finally add these parts. But that gives the whole  $-13$ , right?

In math language,

$$\begin{aligned}\left(\frac{1}{4} \times (-13)\right) + \left(\frac{3}{4} \times (-13)\right) &= \left(\frac{1}{4} + \frac{3}{4}\right) \times (-13) \\ &= 1 \times (-13) \\ &= -13\end{aligned}$$

## Negative times Positive

Imagine water flowing into a tank. Suppose that the water level rises 10 centimeters in every second. We note the water level at a particular time. What will be the change in water level after 2 seconds? It would be  $2 \times 10 = 20$  centimeters above the first mark, right?

What about the water level 2 seconds before our first observation? It would have been 20 centimeters below the first mark.

We can denote “2 seconds before” by  $-2$ . Then the change in water level would be

$$(-2) \times 10 = -20$$

And we can say that  $-2$  centimeters means 20 centimeters below.



**Negative times Negative**

Imagine water flowing out of a tank. And that the water level falls 10 centimeters every second. We measure the water level at a particular time. What would be the change after 2 seconds? It would be 20 centimeters below our first mark, right? What about the water level 2 seconds before? 20 centimeters above the first mark.

Instead of saying that the water level falls 10 centimeters every second, we can say that it changes by  $-20$  centimeters every second. In that case, the change after 2 seconds is

$$2 \times -10 = -20.$$

That is, 20 centimeters below.

If we denote “2 seconds before” by  $-2$  seconds also, the change in water level would be

$$(-2) \times (-10) = 20$$

That is, 20 centimeters above.

Now do these in your head:

- $(12 \times (-7)) + (8 \times (-7))$
- $(29 \times (-2)) + (21 \times (-2))$
- $\left(\frac{2}{7} \times (-17)\right) + \left(\frac{5}{7} \times (-17)\right)$
- $\left(\frac{11}{9} \times (-5)\right) + \left(\frac{7}{9} \times (-5)\right)$
- $(1.5 \times (-9)) + (0.5 \times (-9))$
- $(3.4 \times (-7)) + (0.6 \times (-7))$
- $(4 \times (-0.19)) + (6 \times (-0.19))$

How do we write in algebra, the general principle used here?

$$x(-z) + y(-z) = (x + y)(-z) \text{ for all positive numbers } x, y, z$$

**Another multiplication**

What is  $(6 + (-4)) \times 5$ ?

$6 + (-4) = 6 - 4 = 2$ , isn't it? So,

$$(6 + (-4)) \times 5 = (6 - 4) \times 5 = 2 \times 5 = 10$$

As before, if we multiply 6 and  $-4$  separately by 5 and add, will we get the same answer?

Let's see:

$$6 \times 5 = 30$$

$$(-4) \times 5 = -(4 \times 5) = -20$$

So that

$$(6 \times 5) + (-4) \times 5 = 30 + -20 = 30 - 20 = 10$$

It turns out right, doesn't it?

That is,

$$(6 + (-4)) \times 5 = (6 \times 5) + ((-4) \times 5)$$

Will this be right, if we take  $-5$  instead 5?

Let's check. First let's see what we get when we multiply the sum of 6 and  $-4$  by  $-5$ :

$$(6 + (-4)) \times (-5) = 2 \times (-5) = -10$$

Now we must check whether this is equal to  $(6 \times (-5)) + ((-4) \times (-5))$ .

We have seen that  $6 \times (-5) = -(6 \times 5) = -30$

But what is the meaning of  $(-4) \times (-5)$ ?

We cannot define this as repeated addition; nor can we define the reversed product  $(-5) \times (-4)$  as repeated addition.

So we must give a new meaning to  $(-5) \times (-4)$ .

And this new meaning must be such that  $(6 + (-4)) \times (-5)$  and  $(6 \times (-5)) + ((-4) \times (-5))$  are equal.

We have found the first of these expressions as

$$(6 + (-4)) \times (-5) = -10$$

And in the second expression, we have found the first product

$$6 \times (-5) = -30$$

So,  $(-4) \times (-5)$  added to  $-30$  should give  $-10$ . For this to happen, what should be  $(-4) \times (-5)$ ?

We know that

$$-30 + 20 = -10$$

So, we must *define*

$$(-4) \times (-5) = 20$$

In the same fashion,

$$(-2) \times (-3) = 6$$

$$\left(-\frac{1}{2}\right) \times (-4) = 2$$

$$\left(-\frac{1}{3}\right) \times \left(-\frac{1}{4}\right) = \frac{1}{12}$$

In short we define,

$$(-x) \times (-y) = xy, \text{ for all positive numbers } x, y.$$

Using this we can also see that

$$\frac{-6}{-2} = (-6) \times \left(-\frac{1}{2}\right) = 6 \times \frac{1}{2} = 3$$

### Powers of negatives

What are the powers of 2?

$$2^1 = 2$$

$$2^2 = 2 \times 2 = 4$$

$$2^3 = 4 \times 2 = 8$$

.....

.....

.....

What about the powers of  $-2$ ?

$$(-2)^1 = -2$$

$$(-2)^2 = (-2) \times (-2) = 4$$

$$(-2)^3 = 4 \times (-2) = -8$$

Thus every even power of  $-2$  is equal to the same power of 2; every odd power of  $-2$  is the negative of the same power of 2.

This is true for other numbers also, isn't it?

What are the powers of  $-1$ ?

### Square roots

What is the square root of 25? Since

$$5 \times 5 = 25,$$

a square root of 25 is 5. We have also seen that

$$(-5) \times (-5) = 25$$

So, -5 is also a square root of 25.

Thus any non-zero perfect square has two square roots; one positive and the other negative. The positive square root is denoted by the symbol  $\sqrt{\quad}$ . Thus for example

$$\sqrt{25} = 5$$

The second square root -5 is of course  $-\sqrt{25}$ .

### Let's find out

- $(-2) \times (-5)$
- $(-5) \times (-5)$
- $(-1) \times (-1)$
- $\left(-\frac{1}{3}\right) \times (-2)$
- $\left(-\frac{1}{2}\right) \times \left(-\frac{1}{3}\right)$
- $(-2) \times (-5)$
- $(-12) \div (-4)$
- $(-12) \div (-12)$
- $(-2) \times \left(-\frac{1}{3}\right)$
- $(-3) \div \left(-\frac{1}{2}\right)$
- $(-2) \div \left(-\frac{1}{3}\right)$
- $-2.5 \times -0.2$

### Repeated multiplication

What do we get on simplifying  $(-3) \times 4 \times (-5)$ ?

$$\begin{aligned} (-3) \times 4 \times (-5) &= ((-3) \times 4) \times (-5) \\ &= (-12) \times (-5) \\ &= \dots\dots\dots \end{aligned}$$

We can write this as

$$\begin{aligned} (-3) \times (4 \times (-5)) &= (-3) \times (-20) \\ &= 60 \end{aligned}$$

What about  $(-3) \times (-5) \times 4$ ? Do it yourself.

Now can't you simplify these?

- $(-1) \times 2 \times (-3)$
- $1 \times (-2) \times 3$
- $(-2) \times (-3) \times (-4)$
- $(-1) \times (-1)$
- $(-1)^3$
- $(-1)^5$
- $(-1)^{99}$

Another question:

Using the numbers 3, 4, 5 and their negatives, in how many different ways can we make the product -60?



## Negative of negative

What is  $3 - (-5)$ ?

It means, we have to find out how much is to be added to  $(-5)$  to get 3 (see the section, *Subtraction or addition* in the lesson *Negative Numbers* of class 7).

To get 0 from  $-5$ , we have to add 5. Now to get 3 from 0, we have to add 3. So, to get 3 from  $-5$ , we have to add 5 and 3.

Thus

$$3 - (-5) = 3 + 5 = 8$$

What about  $0 - (-5)$ ?

$$0 - (-5) = 0 + 5 = 5$$

We write  $0 - 5$  as  $-5$ . So, why not write  $0 - (-5)$  as  $-(-5)$ ?

Thus

$$-(-5) = 0 - (-5) = 0 + 5 = 5$$

Similarly,

$$-(-1) = 1$$

$$-\left(-\frac{1}{3}\right) = \frac{1}{3}$$

Let's write this in algebra:

$$-(-x) = x \text{ all positive numbers } x.$$

## The ways of algebra

We write positive numbers such as 1, 2, 3, ... without any sign and negative numbers such as  $-1, -2, -3, \dots$  with the minus sign attached.

Similarly in algebra, we have so far denoted positive numbers using letters such as  $a, b, x, y$  without any sign and negative numbers using letters with minus sign such as  $-a, -b, -x, -y$ .

## Negative as product

What is  $3 \times (-1)$ ?

What is  $(-3) \times (-1)$ ?

We can write

$$(-3) \times (-1) = 3 = -(-3)$$

Thus when we multiply a number, whether positive or negative, by  $(-1)$ , we get its negative. Thus

$$(-1) \times x = -x, \text{ for all numbers } x.$$



## Negative powers

Remember how we computed quotients of powers in Class 7? For example,

$$\frac{4^5}{4^2} = 4^{5-2} = 4^3$$

$$\frac{4^2}{4^5} = \frac{1}{4^{5-2}} = \frac{1}{4^3}$$

Why can't we write:  $\frac{4^2}{4^5} = 4^{2-5} = 4^{-3}$

According to our definition of powers, does  $4^{-3}$  have any meaning? (What is the meaning in saying, the product of  $-3$  fours?)

Just as we gave a new meaning, different from repeated multiplication, to products by negative numbers, so do we give a new meaning to negative powers, different from repeated multiplication.

If we want to get  $\frac{4^2}{4^5} = 4^{2-5} = 4^{-3}$ , then

we should define  $4^{-3} = \frac{1}{4^3}$ . In general,

we make the following definition:

for all  $x \neq 0$  and for all natural numbers  $n$

$$x^{-n} = \frac{1}{x^n}$$

Once we have this definition, we can combine two of the general principles on the quotients of powers to a single

principle, namely  $\frac{x^m}{x^n} = x^{m-n}$  whether

$m > n$  or  $m < n$ .

But in algebra, the usual custom is to denote both positive and negative numbers by letters without signs.

For example, consider the algebraic expression  $x + y$ . If we put  $x = 5$  and  $y = 3$ , we get

$$x + y = 5 + 3 = 8$$

We can also put  $x = 5$  and  $y = -3$  and then we get

$$x + y = 5 + (-3) = 5 - 3 = 2$$

How about  $x = -5$  and  $y = -3$ . In that case,

$$x + y = (-5) + (-3) = -8$$

In the same fashion, if in  $x - y$ , we put  $x = 5, y = 3$ , we get

$$x - y = 5 - 3 = 2$$

And if we put  $x = 5$  and  $y = -3$  in  $x - y$ , we get

$$x - y = 5 - (-3) = 5 + 3 = 8$$

How about  $x = -5, y = -3$ ? Then

$$x - y = (-5) - (-3) = -5 + 3 = -2$$

$-x$  is also an expression. What does it mean? The negative of  $x$ .

Suppose in the expression  $-x$ , we put  $x = 2$ ? We get

$$-x = -2$$

How about  $x = -2$ ?

$$-x = -(-2) = 2$$

Thus in algebra, when we write  $x$  without any sign, it may not be a positive number. And even when we write  $-x$ , it may not be a negative number.

Some algebraic expressions and the values of the letters used in them are given below. Find the values of the expressions in each case:

$x$	$y$	$x + y$	$y + x$	$x - y$	$y - x$	$-x - y$	$-y - x$
7	3	10	10	4	-4	-10	-10
7	-3						
-7	3						
-7	-3						
6	6						
6	-6						
-6	6						
-6	-6						

### General principles

We have seen in Class 7 that the sum of two positive numbers remains the same, even if we change the order in which they are added; for example,  $3 + 5 = 8 = 5 + 3$ .

Do you remember how we stated this as a general principle in the section, *Changing order* of the lesson, *Math Shorthand* of the Class 7 textbook?

$$x + y = y + x, \text{ for all positive numbers } x, y.$$

From the table above, we see that this is true for negative numbers also. So, now we can state this general principle as follows:

$$x + y = y + x, \text{ for all numbers } x, y.$$

Another general principle given in the section Reverse Subtraction of the lesson *Negative Numbers* in Class 7, is this:

$$x - y = -(y - x), \text{ for all positive numbers } x, y \\ \text{with } x < y.$$

In this, there are two conditions on  $x$  and  $y$ : (i)  $x$  and  $y$  should be positive numbers (ii)  $x < y$ . Will this be true even if these conditions are not satisfied?

### Zero power

We have seen that once negative powers are defined suitably, we

would have  $\frac{x^m}{x^n} = x^{m-n}$  whether

$$m > n \text{ or } m < n.$$

One question still remains: what if the exponents are equal? For example,

what is  $\frac{4^5}{4^5}$ ?

Any non-zero number divided by itself gives 1, right? So,

$$\frac{4^5}{4^5} = 1$$

Do we get this if we subtract the exponents?

$4^0$  doesn't have any meaning, does it?

Let's give a new meaning to it also. If

we want  $\frac{4^5}{4^5} = 4^{5-5} = 4^0$ , then we must

define  $4^0 = 1$ .

In fact we make the following general definition:

$$x^0 = 1 \text{ for all } x \neq 0.$$

Once we have this definition also, we need only one general principle for the division of powers:

$$\frac{x^m}{x^n} = x^{m-n}$$

for all  $x \neq 0$  and for all natural numbers  $m$  and  $n$ .

### Merging opposites

Addition and subtraction are opposite operations, right? So are multiplication and division. Once we start using fractions, we can write for any numbers  $x$  and  $y$  with  $y \neq 0$ ,

$$x \div y = x \times \frac{1}{y}$$

Thus division becomes a kind of multiplication.

Similarly, once we start using negative numbers, we can write for any numbers  $x$  and  $y$

$$x - y = x + (-y)$$

and thus subtraction becomes a special case of addition.

In algebraic discussions involving all types of numbers such as fractions and negative numbers, we can use the single form  $xy$  to denote both multiplication and division; and the single form  $x + y$  to denote addition and subtraction.

Complete this table and check:

$x$	$y$	$x - y$	$y - x$	$-(y - x)$
3	5	-2	2	-2
5	3	2	-2	2
5	-3			
3	-5			
-5	3			
3	-5			
-5	-3			
-3	-5			
3	0			
0	3			
-3	0			
0	-3			

Thus we find

$$x - y = -(y - x) \text{ for all numbers } x, y.$$

Some more general principles are given in the lesson *Negative numbers* in the Class 7 textbook. For example,

for all positive numbers  $x, y$

$$-x + x = 0$$

$$-x + y = y - x$$

$$x + (-y) = x - y$$

$$-x + (-y) = -x - y = -(x + y)$$

$$x - (-y) = x + y$$

$$-x - (-y) = -x + y$$

All these are true for any kind of numbers (positive, negative or zero), aren't they?

### Unification

We have seen that

$$(x + y) + z = x + (y + z) \text{ for all positive numbers } x, y, z,$$

in the section, *When there are three* of the chapter *Shorthand Math* of the Class 7 textbook.

Is this also true for all (including negative) numbers?

Complete the table below and check:

$x$	$y$	$z$	$x + y$	$y + z$	$(x + y) + z$	$x + (y + z)$
-5	2	4	-3	6	1	1
5	-2	4				
5	2	-4				
-5	-2	4				
5	-2	-4				
-5	2	-4				
-5	-2	-4				

What do we see from such computations as these?

$$(x + y) + z = x + (y + z), \text{ for all numbers } x, y, z$$

We can see some more things from this. Since the above equation is true for all numbers, it remains true if we replace  $y$  and  $z$  in it by  $-y$  and  $-z$ . That is,

$$(x + (-y)) + (-z) = x + ((-y) + (-z))$$

How do we write this, using the fact that adding a number is the same as subtracting its negative?

$$(x - y) - z = x + (-y - z)$$

Here, we can write  $-y - z$  as  $-(y + z)$ . What do we get then?

$$(x - y) - z = x - (y + z)$$

We have seen this as a general rule of subtraction in Class 7. (The section, *Differences of differences* in the lesson *Shorthand Math*.) Now we see that this is just a special case of the general rule that  $(x + y) + z = x + (y + z)$  for all numbers  $x, y, z$ .

Try to prove the following using this general principle.

### Changing signs

We have seen that for all numbers  $x, y, z$

$$x - (y + z) = x - y - z$$

$$x - (y - z) = x - y + z$$

When we need to subtract a sum or difference in the simplification of algebraic expressions, we use these general principles. We can loosely say that subtraction interchanges the signs  $+$  and  $-$ .

- $(x - y) + z = x - (y - z)$  for all numbers  $x, y, z$ .
- $(x + y) - z = x + (y - z)$  for all numbers  $x, y, z$ .

### Simplification

Remember how we simplified some algebraic expressions in the lesson, *Language of Math* of Class 7?

For example.

$$x + (x + 1) = (x + x) + 1 = 2x + 1$$

What does this mean?

Whatever be the value of  $x$ , if we compute  $x + (x + 1)$  and  $2x + 1$ , the values of these expressions would be equal. And the form  $2x + 1$  is simpler than  $x + (x + 1)$ , isn't it?

Can you simplify  $2x - (x + 1)$  like this?

We have seen that

$$x - (y + z) = (x - y) - z \text{ for all numbers } x, y, z.$$

So, we get

$$2x - (x + 1) = (2x - x) - 1 = x - 1$$

How about  $(3x + y) - (2x + y)$ ?

$$\begin{aligned} (3x + y) - (2x + y) &= (3x + y) - 2x - y \\ &= 3x + y - 2x - y \\ &= (3x - 2x) + (y - y) \\ &= x \end{aligned}$$

One more example: how do we simplify  $(2x + 3y) - (x - 2y)$ ?

We have seen that

$$x - (y - z) = x - y + z \text{ for all numbers } x, y, z.$$

Using this, we find

$$\begin{aligned} (2x + 3y) - (x - 2y) &= (2x + 3y) - x + 2y \\ &= 2x + 3y - x + 2y \\ &= 2x - x + 3y + 2y \\ &= x + 5y \end{aligned}$$

Now try simplifying these:

- $(3x + 2y) - (x + y)$
- $(x + y) - (x - y)$
- $(4x - 3y) - (2x - 5x)$
- $(3x + 2y) - (x - y)$
- $(x - y) - (x + y)$

### Subtraction and division

Just as we have the rules

$$x - (y + z) = (x - y) - z$$

$$x - (y - z) = (x - y) + z$$

for subtraction, we have the following rules for division

$$x \div (y \times z) = (x \div y) \div z$$

$$x \div (y \div z) = (x \div y) \times z$$